

# Steepest Descent

Write Up



To implement the steepest descent algorithm I programmed with [Java](#). I chose to use java for this project due to its simplicity and the convenience of the [Jama](#) library, which handles all of the matrix math. I started this project by building the base algorithm to use to find the steepest descent. The steepest descent algorithm begins by initializing a count which keeps track of the number of iterations until it hits absolute bottom, defined to a specific accuracy. Next I used a do while loop to process the algorithm itself. By using a do while loop, I only had to set up the algorithm once and plug in the users input. After the first step of the algorithm the while portion checks to see if the users input is close enough to the bottom of the curve to stop. If not the loop continues to feed its output back in through the algorithm until it reaches bottom according to the user defined accuracy. Once the program stops, the output is printed out for the user.

## Questions:

$$1. \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} -0.99999 \\ 1.00000 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \text{ x reached in 8 steps}$$

$$x_0 = \begin{bmatrix} 44 \\ 69 \end{bmatrix} \text{ x reached in 10 steps}$$

$$x_0 = \begin{bmatrix} 45 \\ 72 \end{bmatrix} \text{ x reached in 10 steps}$$

$$x_0 = \begin{bmatrix} 105 \\ 2 \end{bmatrix} \text{ x reached in 10 steps}$$

$$\begin{bmatrix} 1.001 & -0.999 \\ -0.999 & 1.001 \end{bmatrix} x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x = \begin{bmatrix} -749.7467959 \\ -750.2467710 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 10 \\ 65 \end{bmatrix} \text{ x reached in 34 steps}$$

$$x_0 = \begin{bmatrix} 15 \\ 66 \end{bmatrix} \text{ x reached in 36 steps}$$

$$x_0 = \begin{bmatrix} 32 \\ 150 \end{bmatrix} \text{ x reached in 16 steps}$$

$$x_0 = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \text{ x reached in 3268 steps}$$

It seems that the output for each different  $x_0$  chosen in wide intervals apart is almost random, but when chosen close together you can see that it follows the altitude of the graph.

2. Five random 10x10 matrices on following pages.

$$A_1 = \begin{bmatrix} 337.0 & 260.0 & 228.0 & 303.0 & 246.0 & 229.0 & 320.0 & 244.0 & 289.0 & 308.0 \\ 260.0 & 243.0 & 196.0 & 223.0 & 202.0 & 203.0 & 255.0 & 181.0 & 216.0 & 244.0 \\ 228.0 & 196.0 & 323.0 & 248.0 & 250.0 & 263.0 & 223.0 & 242.0 & 259.0 & 294.0 \\ 303.0 & 223.0 & 248.0 & 376.0 & 326.0 & 252.0 & 311.0 & 272.0 & 282.0 & 370.0 \\ 246.0 & 202.0 & 250.0 & 326.0 & 314.0 & 240.0 & 263.0 & 227.0 & 239.0 & 344.0 \\ 229.0 & 203.0 & 263.0 & 252.0 & 240.0 & 240.0 & 229.0 & 220.0 & 227.0 & 273.0 \\ 320.0 & 255.0 & 223.0 & 311.0 & 263.0 & 229.0 & 372.0 & 262.0 & 317.0 & 360.0 \\ 244.0 & 181.0 & 242.0 & 272.0 & 227.0 & 220.0 & 262.0 & 291.0 & 288.0 & 291.0 \\ 289.0 & 216.0 & 259.0 & 282.0 & 239.0 & 227.0 & 317.0 & 288.0 & 352.0 & 310.0 \\ 308.0 & 244.0 & 294.0 & 370.0 & 344.0 & 273.0 & 360.0 & 291.0 & 310.0 & 479.0 \end{bmatrix}$$

$$b_1 = \begin{bmatrix} 3.0 \\ 2.0 \\ 8.0 \\ 5.0 \\ 9.0 \\ 7.0 \\ 1.0 \\ 4.0 \\ 9.0 \\ 6.0 \end{bmatrix} \quad x_{0_1} = \begin{bmatrix} 8.0 \\ 4.0 \\ 5.0 \\ 0.0 \\ 3.0 \\ 7.0 \\ 0.0 \\ 5.0 \\ 8.0 \\ 3.0 \end{bmatrix} \quad x_1 = \begin{bmatrix} -21.03851 \\ 22.758378 \\ 31.273297 \\ 30.583319 \\ -17.35853 \\ -43.49098 \\ 9.7042062 \\ 1.7304305 \\ -9.289999 \\ -5.975278 \end{bmatrix}$$

Total number of iterations was:89210

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$$A_2 = \begin{bmatrix} 338.0 & 226.0 & 176.0 & 205.0 & 220.0 & 253.0 & 244.0 & 223.0 & 156.0 & 280.0 \\ 226.0 & 295.0 & 178.0 & 154.0 & 173.0 & 236.0 & 244.0 & 266.0 & 183.0 & 250.0 \\ 176.0 & 178.0 & 284.0 & 247.0 & 198.0 & 207.0 & 185.0 & 222.0 & 149.0 & 265.0 \\ 205.0 & 154.0 & 247.0 & 295.0 & 177.0 & 211.0 & 151.0 & 210.0 & 165.0 & 288.0 \\ 220.0 & 173.0 & 198.0 & 177.0 & 27.0 & 217.0 & 185.0 & 208.0 & 124.0 & 232.0 \\ 253.0 & 236.0 & 207.0 & 211.0 & 217.0 & 281.0 & 249.0 & 252.0 & 184.0 & 273.0 \\ 244.0 & 244.0 & 185.0 & 151.0 & 185.0 & 249.0 & 288.0 & 210.0 & 152.0 & 227.0 \\ 223.0 & 266.0 & 222.0 & 210.0 & 208.0 & 252.0 & 210.0 & 309.0 & 183.0 & 298.0 \\ 156.0 & 183.0 & 149.0 & 165.0 & 124.0 & 184.0 & 152.0 & 183.0 & 241.0 & 254.0 \\ 280.0 & 250.0 & 265.0 & 288.0 & 232.0 & 273.0 & 227.0 & 298.0 & 254.0 & 422.0 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 8.0 \\ 8.0 \\ 2.0 \\ 2.0 \\ 3.0 \\ 7.0 \\ 2.0 \\ 0.0 \\ 5.0 \\ 8.0 \end{bmatrix} \quad x_{0_2} = \begin{bmatrix} 9.0 \\ 3.0 \\ 6.0 \\ 3.0 \\ 1.0 \\ 8.0 \\ 1.0 \\ 9.0 \\ 5.0 \\ 1.0 \end{bmatrix} \quad x_2 = \begin{bmatrix} -0.29806 \\ -0.56465 \\ -0.79175 \\ 0.731233 \\ 0.716686 \\ -1.34044 \\ 1.064291 \\ 0.811893 \\ 0.346313 \\ -0.36966 \end{bmatrix}$$

Total number of iterations was:3359

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$$A_3 = \begin{bmatrix} 340.0 & 169.0 & 315.0 & 165.0 & 274.0 & 276.0 & 176.0 & 281.0 & 263.0 & 220.0 \\ 169.0 & 189.0 & 154.0 & 147.0 & 117.0 & 189.0 & 104.0 & 192.0 & 120.0 & 193.0 \\ 315.0 & 154.0 & 386.0 & 163.0 & 295.0 & 269.0 & 164.0 & 266.0 & 282.0 & 254.0 \\ 165.0 & 147.0 & 163.0 & 154.0 & 158.0 & 172.0 & 109.0 & 187.0 & 139.0 & 215.0 \\ 274.0 & 117.0 & 295.0 & 158.0 & 345.0 & 271.0 & 159.0 & 231.0 & 289.0 & 250.0 \\ 276.0 & 189.0 & 269.0 & 172.0 & 271.0 & 367.0 & 170.0 & 303.0 & 261.0 & 271.0 \\ 176.0 & 104.0 & 164.0 & 109.0 & 159.0 & 170.0 & 145.0 & 156.0 & 135.0 & 183.0 \\ 281.0 & 192.0 & 266.0 & 187.0 & 231.0 & 303.0 & 156.0 & 317.0 & 242.0 & 242.0 \\ 263.0 & 120.0 & 282.0 & 139.0 & 289.0 & 261.0 & 135.0 & 242.0 & 277.0 & 184.0 \\ 220.0 & 193.0 & 254.0 & 215.0 & 250.0 & 271.0 & 183.0 & 242.0 & 184.0 & 389.0 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 1.0 \\ 3.0 \\ 3.0 \\ 9.0 \\ 8.0 \\ 3.0 \\ 4.0 \\ 5.0 \\ 1.0 \\ 5.0 \end{bmatrix} \quad x_{0_3} = \begin{bmatrix} 2.0 \\ 8.0 \\ 6.0 \\ 5.0 \\ 2.0 \\ 4.0 \\ 5.0 \\ 9.0 \\ 1.0 \\ 8.0 \end{bmatrix} \quad x_3 = \begin{bmatrix} 4.65841 \\ 3.18008 \\ -7.0556 \\ -14.588 \\ -9.8738 \\ -6.6798 \\ -2.9020 \\ 0.06889 \\ 18.7439 \\ 11.8999 \end{bmatrix}$$

Total number of iterations was:25582

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$$A_4 = \begin{bmatrix} 320.0 & 191.0 & 336.0 & 199.0 & 221.0 & 287.0 & 153.0 & 203.0 & 231.0 & 245.0 \\ 191.0 & 191.0 & 201.0 & 151.0 & 150.0 & 205.0 & 132.0 & 127.0 & 115.0 & 147.0 \\ 336.0 & 201.0 & 397.0 & 229.0 & 270.0 & 293.0 & 174.0 & 232.0 & 243.0 & 254.0 \\ 199.0 & 151.0 & 229.0 & 253.0 & 166.0 & 159.0 & 105.0 & 176.0 & 139.0 & 134.0 \\ 221.0 & 150.0 & 270.0 & 166.0 & 244.0 & 209.0 & 137.0 & 129.0 & 119.0 & 170.0 \\ 287.0 & 205.0 & 293.0 & 159.0 & 209.0 & 314.0 & 169.0 & 194.0 & 218.0 & 205.0 \\ 153.0 & 132.0 & 174.0 & 105.0 & 137.0 & 169.0 & 175.0 & 87.0 & 123.0 & 76.0 \\ 203.0 & 127.0 & 232.0 & 176.0 & 129.0 & 194.0 & 87.0 & 243.0 & 244.0 & 158.0 \\ 231.0 & 115.0 & 243.0 & 139.0 & 119.0 & 218.0 & 123.0 & 244.0 & 306.0 & 175.0 \\ 245.0 & 147.0 & 254.0 & 134.0 & 170.0 & 205.0 & 76.0 & 158.0 & 175.0 & 273.0 \end{bmatrix}$$

$$b_4 = \begin{bmatrix} 1.0 \\ 3.0 \\ 5.0 \\ 1.0 \\ 8.0 \\ 8.0 \\ 1.0 \\ 3.0 \\ 8.0 \\ 4.0 \end{bmatrix} \quad x_{0_4} = \begin{bmatrix} 9.0 \\ 2.0 \\ 3.0 \\ 5.0 \\ 4.0 \\ 8.0 \\ 1.0 \\ 5.0 \\ 7.0 \\ 1.0 \end{bmatrix} \quad x_4 = \begin{bmatrix} 52.16077 \\ -13.5676 \\ -29.3920 \\ -21.8550 \\ 2.049248 \\ -24.5567 \\ 30.14648 \\ 57.27822 \\ -43.1938 \\ 1.863528 \end{bmatrix}$$

Total number of iterations was:292053

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$$A_5 = \begin{bmatrix} 178.0 & 155.0 & 155.0 & 137.0 & 135.0 & 228.0 & 103.0 & 82.0 & 194.0 & 130.0 \\ 155.0 & 243.0 & 169.0 & 174.0 & 134.0 & 167.0 & 112.0 & 101.0 & 190.0 & 134.0 \\ 155.0 & 169.0 & 293.0 & 171.0 & 232.0 & 264.0 & 156.0 & 98.0 & 273.0 & 196.0 \\ 137.0 & 174.0 & 171.0 & 209.0 & 150.0 & 214.0 & 124.0 & 106.0 & 159.0 & 183.0 \\ 135.0 & 134.0 & 232.0 & 150.0 & 255.0 & 187.0 & 119.0 & 86.0 & 212.0 & 147.0 \\ 228.0 & 167.0 & 264.0 & 214.0 & 187.0 & 404.0 & 159.0 & 116.0 & 280.0 & 235.0 \\ 103.0 & 112.0 & 156.0 & 124.0 & 119.0 & 159.0 & 159.0 & 66.0 & 142.0 & 141.0 \\ 82.0 & 101.0 & 98.0 & 106.0 & 86.0 & 116.0 & 66.0 & 84.0 & 104.0 & 92.0 \\ 194.0 & 190.0 & 273.0 & 159.0 & 212.0 & 280.0 & 142.0 & 104.0 & 302.0 & 178.0 \\ 130.0 & 134.0 & 196.0 & 183.0 & 147.0 & 235.0 & 141.0 & 92.0 & 178.0 & 201.0 \end{bmatrix}$$

$$b_5 = \begin{bmatrix} 5.0 \\ 1.0 \\ 2.0 \\ 6.0 \\ 4.0 \\ 4.0 \\ 4.0 \\ 7.0 \\ 0.0 \\ 6.0 \end{bmatrix} \quad x_{0_5} = \begin{bmatrix} 2.0 \\ 1.0 \\ 0.0 \\ 9.0 \\ 9.0 \\ 7.0 \\ 6.0 \\ 3.0 \\ 5.0 \\ 8.0 \end{bmatrix} \quad x_5 = \begin{bmatrix} -1.23711 \\ 0.634536 \\ -1.19294 \\ -0.49543 \\ 0.584462 \\ 0.656706 \\ 0.139678 \\ -0.24580 \\ 0.573776 \\ 0.272740 \end{bmatrix}$$

Total number of iterations was:8486

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