

Jacobi Algorithm: Write Up

To implement the Jacobi algorithm I programmed with [Java](#). I chose to use java for this project due to its simplicity and the convenience of the [Jama](#) library, which handles all of the matrix math. In the program I offered several options to customize both the composition of the matrix and the method of running the Jacobi algorithm. I offer a way to either use searching or to skip the searching step and systematically create the sub matrix. I also offer the ability to save one matrix at a time. This way I could compare the same matrix with searching and without. This capability demonstrates, with out graphing, how much simpler the searching method makes the algorithm. I also added a timer to keep track of the time it takes to reach a diagonal matrix. Originally the intent was to graph that against the data farmed out of the Jacobi application, but with such small matrices the differences in times are negligible and could be attributed to other miscellaneous programs running on the computer at the same time.

Once I graphed all the output data through a third party graphing program ([Microsoft® Excel](#)), I was able to clearly see the trends present between the two methods. Some characteristics I noticed were that the searching method never exceeds the theoretical bound, where as it were a normal occurrence for the method without searching to jump out of bounds. The searching method always moves in a downward curve fashion, meaning that each step is definitely getting closer to a diagonal matrix, but the method without searching move very erratically. There are several instances where the graph of the no searching levels of, which means the algorithm has no effect on the matrix, probably because those values were already zeroed out. There are also a few instances where the graph actually curves upward, meaning the matrix is actually becoming less diagonal. Simply by looking at the graphs, you can see that the searching method curves follow much closer to an exponential model than those of the non searching method (figure 1).

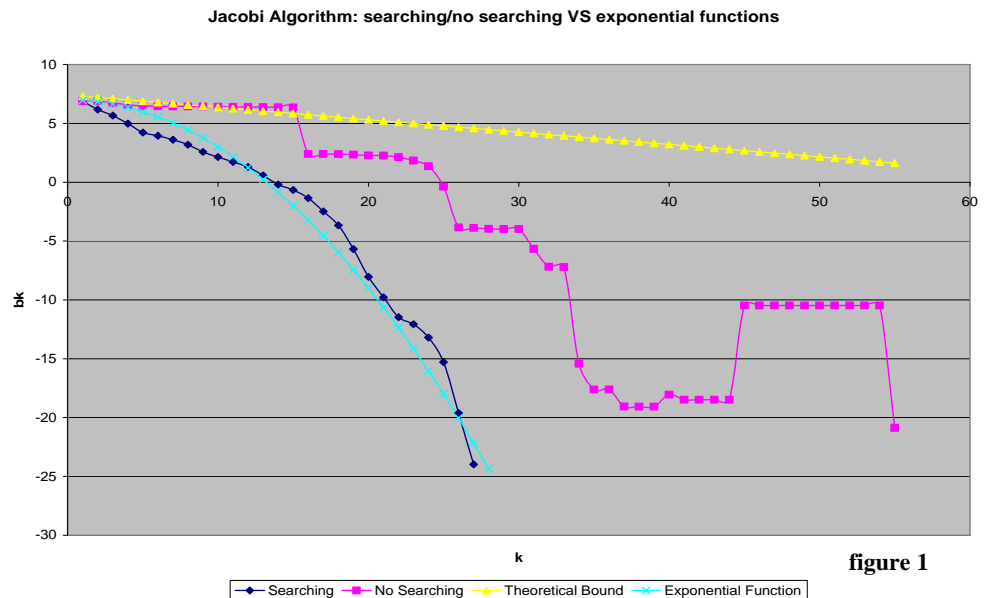


figure 1

Computing the Jacobi Algorithm using the searching method would be the best implementation. The searching method will always take either less than or equal to the number of iterations as used by the non searching method. By searching you eradicate the possibility of wasting iterations not improving conditions in the matrix. The computing power it takes to run through the matrix and find a max value versus the power taken by the wasted iterations is comparable. Depending on how many time the program is forced to iterate through the matrix in the non searching method, the searching method can be much more effective and therefore is the best choice when implementing the Jacobi algorithm.

Matrix 1 Results

Matrix A:

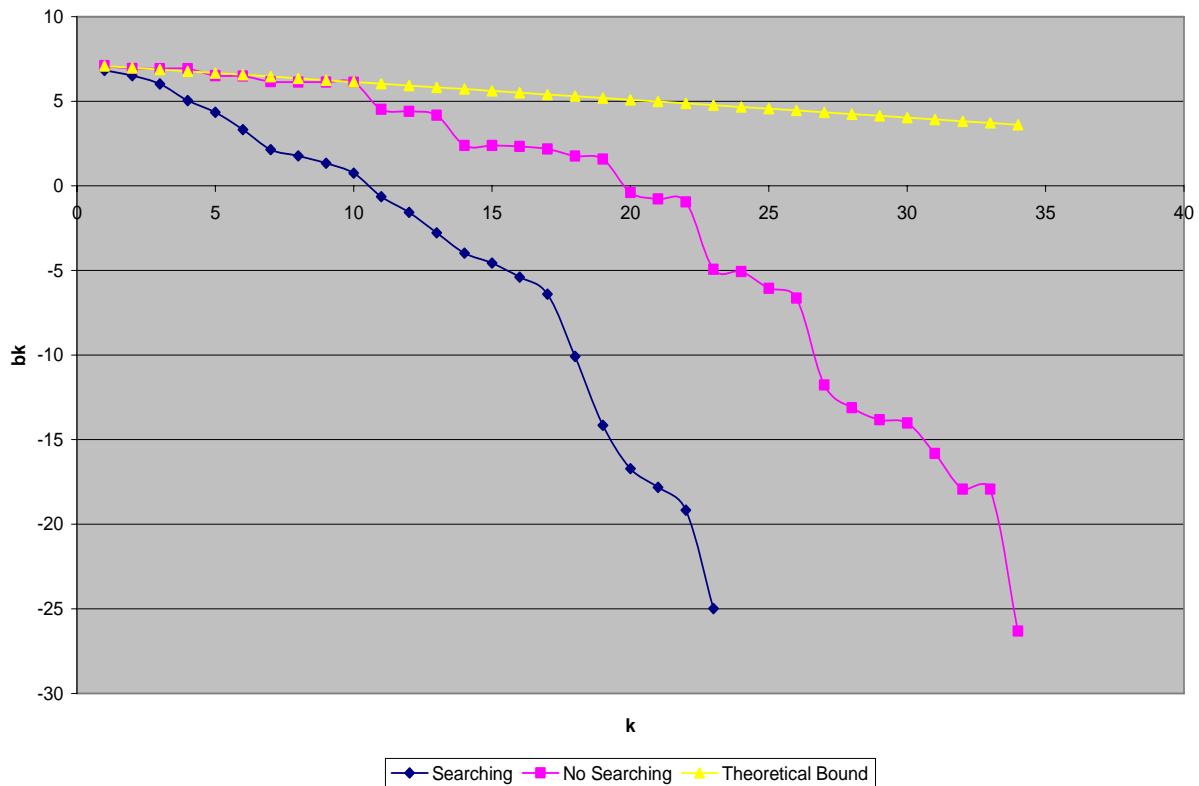
$$\begin{bmatrix} -14.91 & -5.46 & -11.04 & 7.59 & -9.51 \\ -5.46 & -15.51 & -7.52 & 1.08 & 14.32 \\ -11.04 & -7.52 & 15.15 & -9.56 & 0.94 \\ 7.59 & 1.08 & -9.56 & -19.07 & 3.13 \\ -9.51 & 14.32 & 0.94 & 3.13 & -0.37 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 22.81 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 12.52 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -16.43 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -26.07 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -27.55 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 23	Total Iterations: 34
Total Time: 0.00030 seconds	Total Time: 0.0010 seconds

Jacobian Algorithm Comparison 1



Matrix 1 Data

k	$b_k(\text{Searching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	6.823365693932764	7.103315172412159	7.086788348647603
2	6.515117784137418	6.940461013493451	6.981427832989776
3	6.023483041325357	6.922831299613034	6.87606731733195
4	5.039491076664045	6.920565295938993	6.770706801674123
5	4.3435015329810716	6.5083525614832975	6.665346286016297
6	3.3282167090024686	6.48539215626051	6.5599857703584705
7	2.1411082346277266	6.159487088666889	6.454625254700645
8	1.7760995612394008	6.128866163600248	6.349264739042818
9	1.3350702729609707	6.128784020502465	6.243904223384992
10	0.75573634557932	6.123553772119119	6.138543707727166
11	-0.6421061656527705	4.520674839265441	6.033183192069339
12	-1.5681689087566055	4.396938592023689	5.927822676411513
13	-2.783145958812042	4.162845188604271	5.822462160753687
14	-3.976979103119229	2.3871204391760608	5.717101645095861
15	-4.568431635221972	2.3843638703535657	5.611741129438034
16	-5.390912122771273	2.3311972294064063	5.506380613780208
17	-6.3968839275518175	2.17974692044864	5.401020098122382
18	-10.078508689607332	1.7578494475977295	5.295659582464555
19	-14.154380272966623	1.581183296782349	5.190299066806729
20	-16.717309118298452	-0.3911888694681286	5.084938551148903
21	-17.821020395783577	-0.7812040949265843	4.979578035491077
22	-19.16228066219234	-0.9555234744272886	4.87421751983325
23	-24.987670141491247	-4.9355220045414425	4.768857004175424
24		-5.068592512202663	4.663496488517598
25		-6.059847165246598	4.5581359728597715
26		-6.631567148260095	4.452775457201945
27		-11.76838114475045	4.347414941544119
28		-13.123444018919225	4.242054425886293
29		-13.827148618105497	4.136693910228466
30		-14.021114280765937	4.03133339457064
31		-15.823216285765188	3.925972878912814
32		-17.931405168862405	3.8206123632549875
33		-17.934908334847734	3.715251847597161
34		-26.3219058748695	3.609891331939335

Matrix 2 Results

Matrix A:

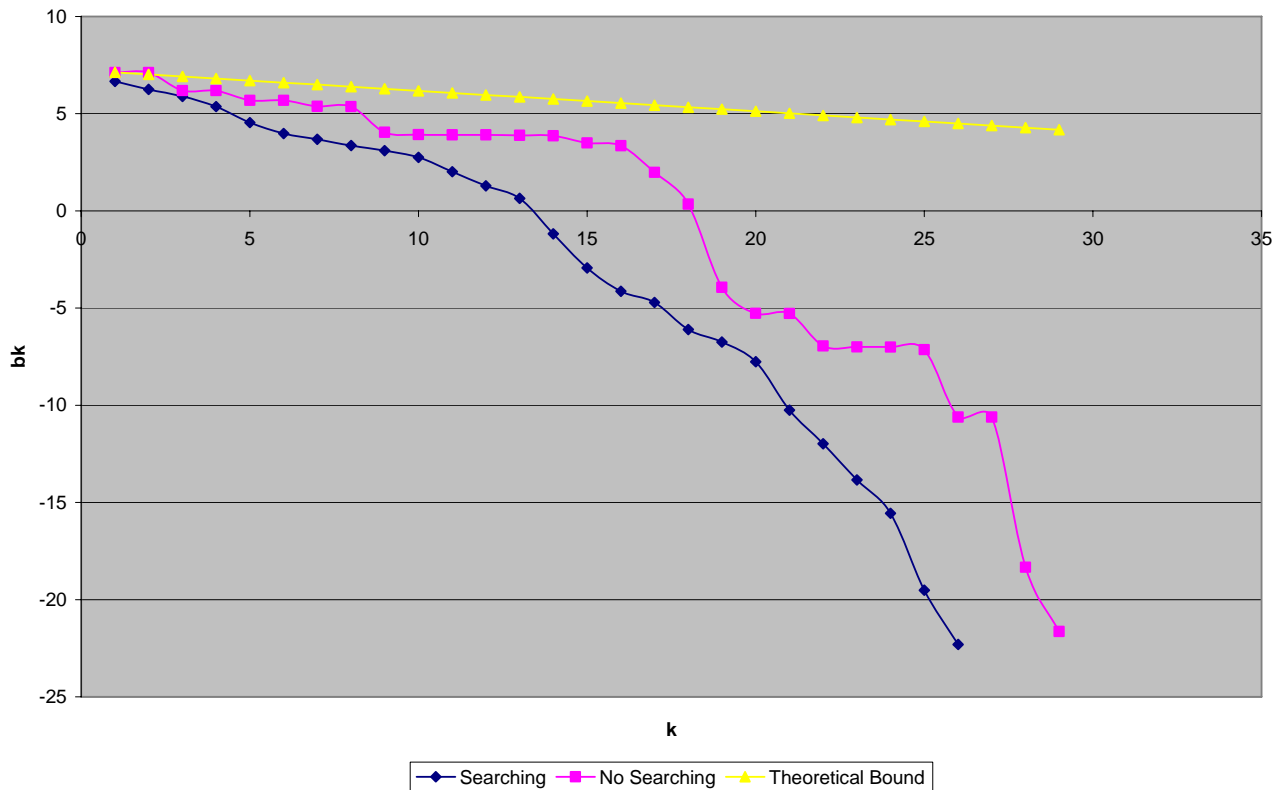
$$\begin{bmatrix} 7.97 & 8.41 & -3.14 & 17.22 & 5.37 \\ 8.41 & -5.11 & 9.51 & 8.09 & 4.02 \\ -3.14 & 9.51 & 5.59 & 3.58 & 7.05 \\ 17.22 & 8.09 & 3.58 & 14.97 & -6.96 \\ 5.37 & 4.02 & 7.05 & -6.96 & 0.29 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 33.10 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 14.42 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 4.26 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -12.09 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -15.98 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 26	Total Iterations: 29
Total Time: < 0.00001 seconds	Total Time: < 0.00001 seconds

Jacobian Algorithm Comparison 2



Matrix 2 Data

k	$b_k(\text{Searchching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	6.665599924541685	7.120137947787079	7.123196674413394
2	6.24679990687101	7.117131737403942	7.017836158755567
3	5.884576895255498	6.180843665009374	6.912475643097741
4	5.360054155161014	6.180512879619857	6.807115127439914
5	4.538393391367721	5.683991105443231	6.701754611782088
6	3.975337955807293	5.678686582568212	6.5963940961242615
7	3.677998777582785	5.380577916923609	6.491033580466436
8	3.358688793468009	5.362671513565186	6.385673064808609
9	3.095291652597692	4.036428143333595	6.280312549150783
10	2.748235516385778	3.9174906855514697	6.174952033492957
11	2.007398294075773	3.9124858803667375	6.06959151783513
12	1.286241891574443	3.912264439268782	5.964231002177304
13	0.6443370993691516	3.8815886922649785	5.858870486519478
14	-1.1807129334007642	3.86546271151103	5.753509970861652
15	-2.936303688599798	3.4882333638113936	5.648149455203825
16	-4.1406249171322465	3.345330453866361	5.542788939545999
17	-4.708761499988823	1.9822786974849282	5.437428423888173
18	-6.102296233978596	0.3388773649826339	5.332067908230346
19	-6.7542513288331945	-3.935289958353647	5.22670739257252
20	-7.7602997066346875	-5.269746635709455	5.121346876914694
21	-10.251399604912296	-5.284258760322603	5.015986361256868
22	-11.987146087657608	-6.951444919090146	4.910625845599041
23	-13.839646349306195	-6.997929388607699	4.805265329941215
24	-15.560256223683753	-7.010888857756963	4.699904814283389
25	-19.522306439808823	-7.145557783504921	4.5945442986255625
26	-22.300465504210035	-10.607647901945956	4.489183782967736
27		-10.608728974616133	4.38382326730991
28		-18.335521041282078	4.278462751652084
29		-21.64397834586285	4.173102235994257

Matrix 3 Results

Matrix A:

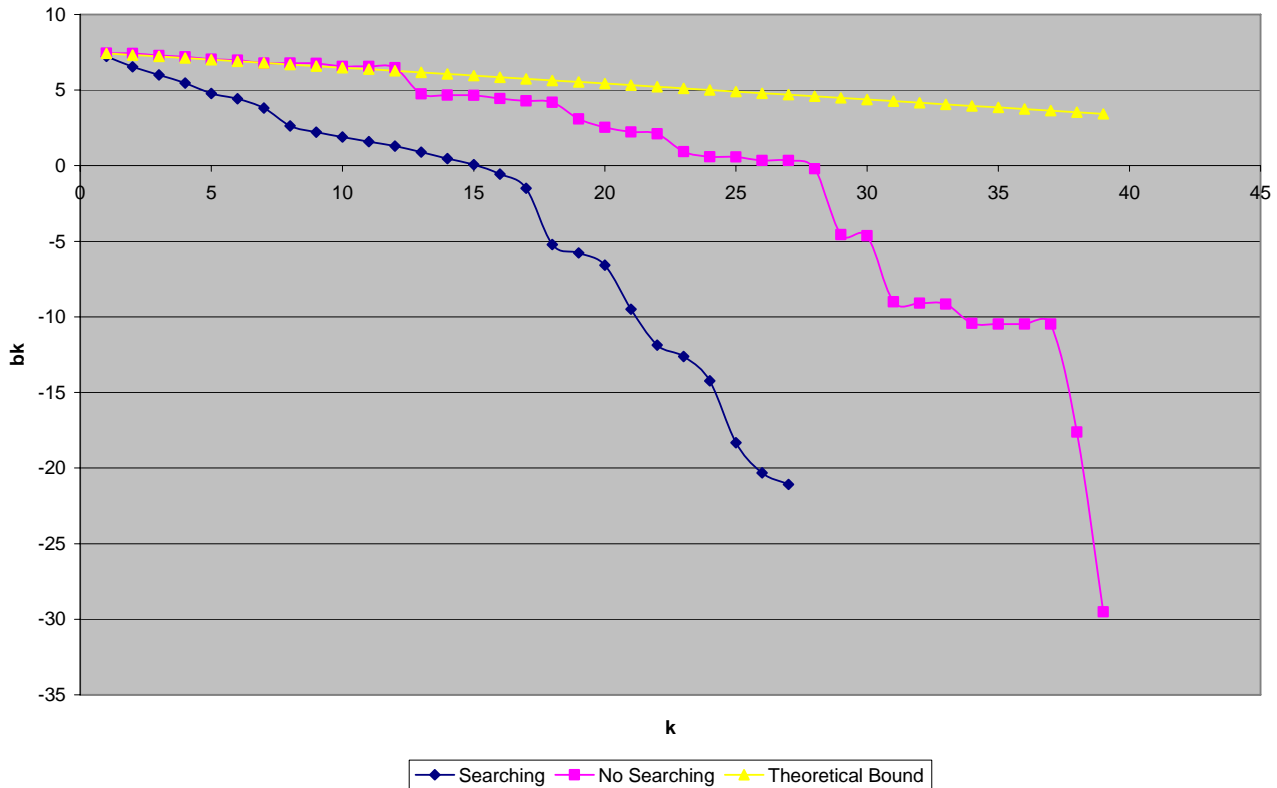
$$\begin{bmatrix} -5.31 & -7.87 & -15.40 & -5.50 & -15.71 \\ -7.87 & 13.56 & -1.01 & 7.85 & 9.28 \\ -15.40 & -1.01 & -16.68 & -9.24 & -11.20 \\ -5.50 & 7.85 & -9.24 & -0.02 & -0.94 \\ -15.71 & 9.28 & -11.20 & -0.94 & -10.15 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 25.93 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 5.95 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -3.29 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -5.77 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -41.41 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 27	Total Iterations: 39
Total Time: < 0.00001 seconds	Total Time: 0.0010 seconds

Jacobian Algorithm Comparison 3



Matrix 3 Data

k	$b_k(\text{Searchching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	7.22946078844076	7.439874968074716	7.430035552097454
2	6.540701473417298	7.42372254791086	7.324675036439627
3	6.001470378533812	7.2878338177173845	7.2193145207818015
4	5.4580377502160555	7.1993534336547205	7.113954005123975
5	4.774487198204759	7.042626289611125	7.008593489466149
6	4.4164867075431635	6.97861838373165	6.903232973808322
7	3.8129179471549914	6.775118560708819	6.797872458150496
8	2.6233300173617575	6.773771238048913	6.69251194249267
9	2.2081091196180287	6.763816689786399	6.5871514268348434
10	1.8881762796654	6.573458582524197	6.481790911177017
11	1.580420065833965	6.559954659142974	6.376430395519191
12	1.2885683138923494	6.479610958376572	6.271069879861365
13	0.8979455610490054	4.738823977342718	6.165709364203538
14	0.4688675693019381	4.669987353453894	6.060348848545712
15	0.06934587518762468	4.651125236652165	5.954988332887886
16	-0.5618756181301685	4.438611653796364	5.8496278172300595
17	-1.494245046528479	4.284460486787832	5.744267301572233
18	-5.2215093771549155	4.194607291279269	5.638906785914407
19	-5.777801211837107	3.0891550059202415	5.533546270256581
20	-6.576315083372274	2.528135230288184	5.428185754598754
21	-9.499355129427716	2.2334056673338734	5.322825238940928
22	-11.86865584163141	2.100531670887677	5.217464723283102
23	-12.629035903799709	0.9226881431531604	5.112104207625276
24	-14.229703808279673	0.5940375515087719	5.006743691967449
25	-18.31908360876543	0.5761581161475593	4.901383176309623
26	-20.310080669020266	0.34509705461279966	4.796022660651797
27	-21.086517707700867	0.3415288152299453	4.69066214499397
28		-0.2140504768887498	4.585301629336144
29		-4.566098543365602	4.4799411136783185
30		-4.630339121416258	4.374580598020492
31		-9.000869430391418	4.269220082362665
32		-9.099039841267704	4.163859566704839
33		-9.153387325020908	4.058499051047013
34		-10.427034349891327	3.9531385353891864
35		-10.470322268672996	3.84777801973136
36		-10.479318956724743	3.7424175040735337
37		-10.479319370283937	3.637056988415708
38		-17.618081326024573	3.531696472757881
39		-29.514046366709046	3.426335957100055

Matrix 4 Results

Matrix A:

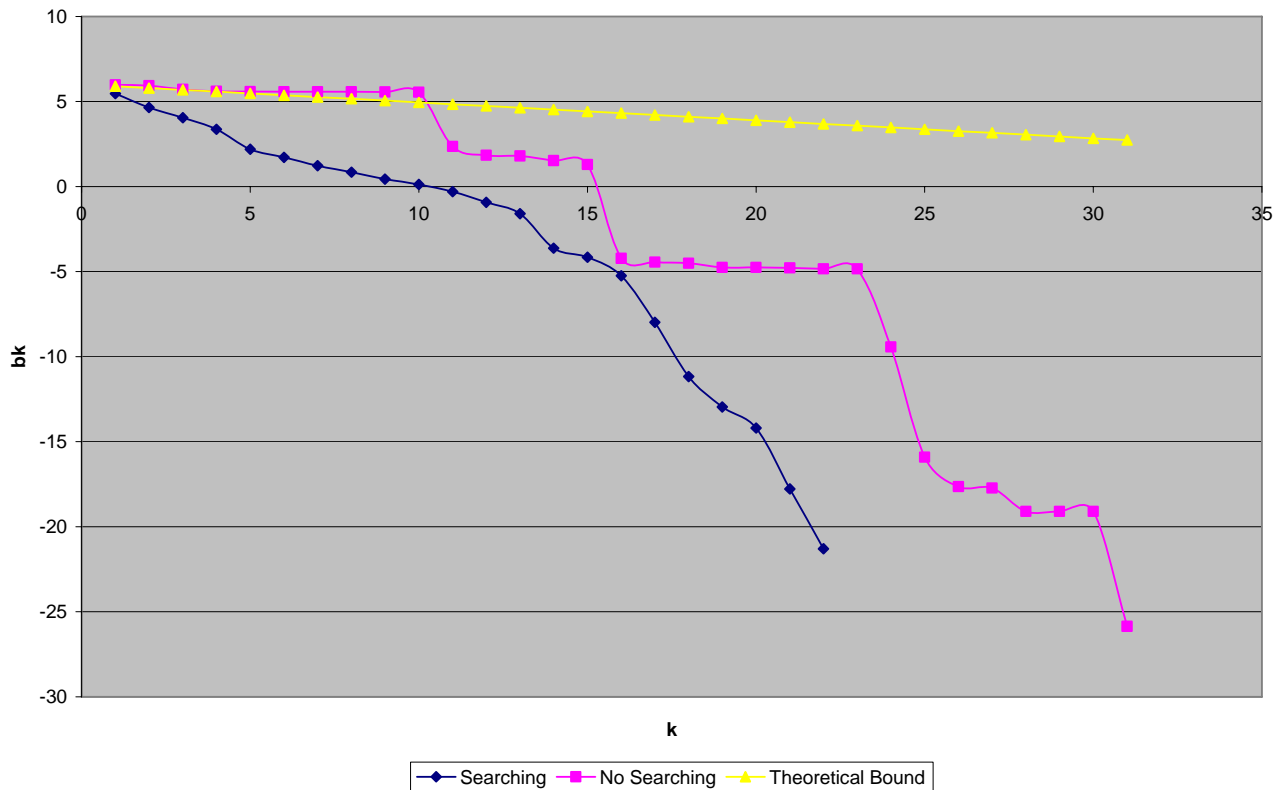
$$\begin{bmatrix} 12.60 & 5.84 & 2.74 & 0.77 & -9.16 \\ 5.84 & 9.85 & 1.79 & 3.94 & 5.81 \\ 2.74 & 1.79 & -3.93 & -3.50 & -1.18 \\ 0.77 & 3.94 & -3.50 & -16.85 & 3.10 \\ -9.16 & 5.81 & -1.18 & 3.10 & 5.55 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 19.74 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 14.68 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -3.38 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -5.22 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -18.60 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 22	Total Iterations: 31
Total Time: 0.0010 seconds	Total Time: 0.0010 seconds

Jacobian Algorithm Comparison 4



Matrix 4 Data

k	$b_k(\text{Searching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	5.4629998255475	5.984991346685516	5.895549155752682
2	4.650996861653789	5.922722058380378	5.790188640094855
3	4.041993017418543	5.711465706287549	5.684828124437029
4	3.366268938160658	5.603166401025313	5.579467608779202
5	2.198737380402182	5.579285860475276	5.474107093121376
6	1.7136464019582769	5.577394083494243	5.36874657746355
7	1.221772386721722	5.577366429691485	5.263386061805724
8	0.8456967969517252	5.5770121356358935	5.158025546147897
9	0.43458253354016696	5.554928700070461	5.052665030490071
10	0.12696136494574203	5.547631005684301	4.947304514832245
11	-0.2949721870495824	2.3637085044428052	4.841943999174418
12	-0.9283476729758492	1.8330188191361465	4.7365834835165925
13	-1.5975551557574097	1.8003826000501817	4.631222967858766
14	-3.627181611124886	1.5321814805044398	4.52586245220094
15	-4.151875372338122	1.2912605374072694	4.420501936543113
16	-5.248709879265744	-4.225983999720704	4.315141420885287
17	-7.98413512820588	-4.4360852861765325	4.209780905227461
18	-11.174006721355733	-4.511597647346914	4.1044203895696345
19	-12.953063734550941	-4.7523110534845685	3.999059873911808
20	-14.20295804568824	-4.756924142625609	3.893699358253982
21	-17.7757979533312	-4.775668099028817	3.788338842596156
22	-21.294514239234413	-4.83700621098837	3.6829783269383296
23		-4.838388617684816	3.5776178112805033
24		-9.421517917272206	3.472257295622677
25		-15.913178464496857	3.3668967799648506
26		-17.6371289216364	3.2615362643070243
27		-17.723135771766728	3.156175748649198
28		-19.099485659949977	3.050815232991372
29		-19.10298447388762	2.9454547173335457
30		-19.10298447388762	2.8400942016757194
31		-25.858869872991686	2.734733686017893

Matrix 5 Results

Matrix A:

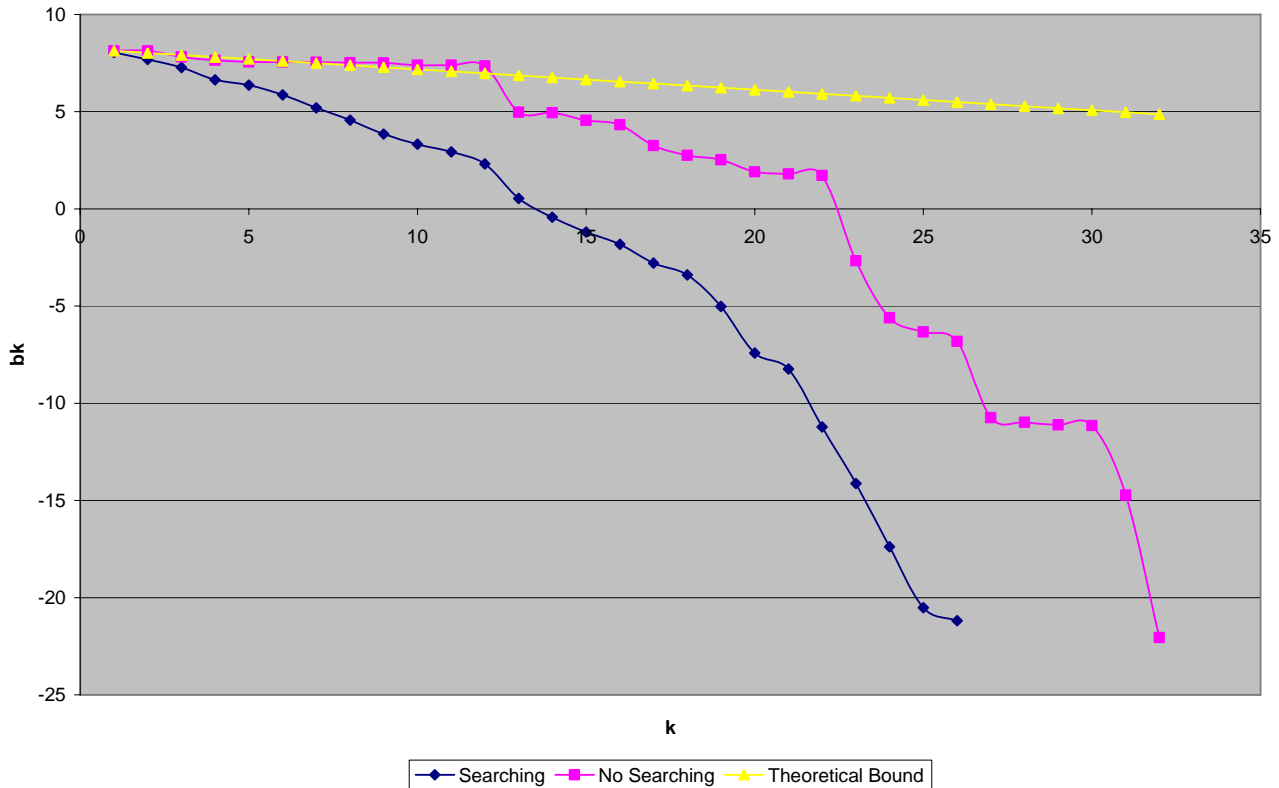
$$\begin{bmatrix} 13.57 & 17.90 & -0.54 & -16.45 & 18.06 \\ 17.90 & -7.47 & -13.52 & 9.62 & -15.25 \\ -0.54 & -13.52 & -12.61 & 10.54 & 16.04 \\ -16.45 & 9.62 & 10.54 & -8.36 & 9.42 \\ 18.06 & -15.25 & 16.04 & 9.42 & -15.58 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 29.13 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 18.36 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.94 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -30.02 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -48.86 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 26	Total Iterations: 32
Total Time: 0.0010 seconds	Total Time: 0.0010 seconds

Jacobian Algorithm Comparison 5



Matrix 5 Data

k	$b_k(\text{Searching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	8.042900931336325	8.131142857913169	8.127892842201009
2	7.691396020896066	8.1308645610963	8.022532326543184
3	7.267180518155416	7.816706389103205	7.917171810885357
4	6.6367548653970845	7.638525748011024	7.81181129522753
5	6.35950127960651	7.556530238468729	7.706450779569704
6	5.862815769311059	7.551366331342905	7.601090263911877
7	5.195230789093966	7.549657336099509	7.4957297482540515
8	4.565352313563807	7.514743380445237	7.390369232596225
9	3.84751937739977	7.514103284966699	7.285008716938399
10	3.3219204295337046	7.386652324553754	7.179648201280573
11	2.9275619715219454	7.385018879959551	7.074287685622746
12	2.3113532244821147	7.353654568415492	6.96892716996492
13	0.5341968121475825	4.966371004469881	6.8635666543070935
14	-0.43194840103187326	4.947624620733873	6.758206138649268
15	-1.1956610724852343	4.552457615894569	6.652845622991441
16	-1.8298762526112307	4.326455133874549	6.547485107333615
17	-2.7902476017055315	3.2575854191524987	6.442124591675789
18	-3.3978311782635524	2.7499110030705376	6.336764076017962
19	-5.021477045501134	2.5258711624900343	6.231403560360135
20	-7.413745334465717	1.909319836283983	6.1260430447023095
21	-8.239666714844207	1.7985515470547877	6.020682529044484
22	-11.21407315306084	1.7110779788884543	5.915322013386657
23	-14.12617843496129	-2.671087323978891	5.809961497728831
24	-17.382095632175755	-5.604412612967772	5.704600982071005
25	-20.513390162546006	-6.330962704751855	5.599240466413178
26	-21.1858433341441	-6.82113497350348	5.4938799507553515
27		-10.74027319410427	5.388519435097526
28		-10.975962569819012	5.2831589194397
29		-11.108871860236485	5.177798403781873
30		-11.153019106884217	5.072437888124047
31		-14.726866162068069	4.967077372466221
32		-22.050301818494276	4.861716856808394

Matrix 6 Results

Matrix A:

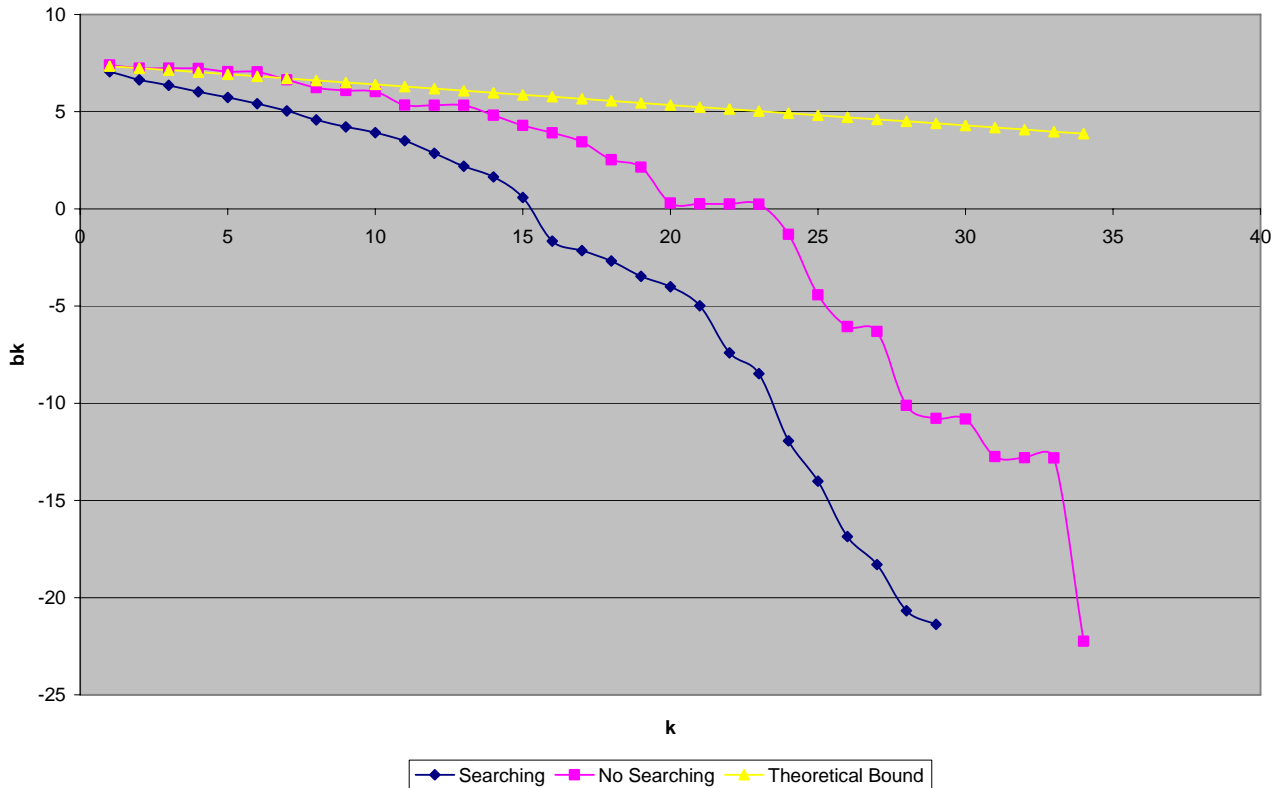
$$\begin{bmatrix} 14.34 & -10.81 & 7.78 & -16.66 & -10.30 \\ -10.81 & -18.73 & 6.69 & -0.24 & -1.22 \\ 7.78 & 6.69 & 7.12 & -11.34 & 4.63 \\ -16.66 & -0.24 & -11.34 & -2.54 & 10.11 \\ -10.30 & -1.22 & 4.63 & 10.11 & -4.39 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 34.82 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 10.14 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -4.79 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -18.39 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -25.99 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 29	Total Iterations: 34
Total Time: < 0.00001 seconds	Total Time: < 0.00001 seconds

Jacobian Algorithm Comparison 6



Matrix 6 Data

k	$b_k(\text{Searchching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	7.059389796183948	7.396000460011884	7.344059788382151
2	6.644155640160603	7.232859303948544	7.2386992727243245
3	6.3494942200760125	7.228747219283216	7.133338757066499
4	6.013928915704803	7.222429531321929	7.027978241408672
5	5.72980518576171	7.058419504483913	6.922617725750846
6	5.407849297987386	7.045965863992716	6.817257210093019
7	5.038029891181048	6.643948599864022	6.711896694435193
8	4.578847250810107	6.238203454330582	6.606536178777366
9	4.223312619135448	6.084626373856184	6.5011756631195405
10	3.9228066965180304	6.029327429695455	6.395815147461715
11	3.5037378136176414	5.3410340940072425	6.290454631803888
12	2.8548576423443817	5.327723241280781	6.185094116146062
13	2.1913950375448543	5.327614134283876	6.079733600488235
14	1.6403867132984375	4.813110627238324	5.974373084830409
15	0.5773688597158142	4.293933395694189	5.8690125691725825
16	-1.6618075152107945	3.908971369583768	5.763652053514757
17	-2.1439760808802855	3.438922118711055	5.658291537856931
18	-2.685832098509537	2.524909296987085	5.552931022199104
19	-3.4733667644692217	2.1448203783541935	5.447570506541277
20	-4.00697276695736	0.29234665679102323	5.342209990883451
21	-4.985817064304015	0.2618330907551604	5.236849475225625
22	-7.3989695698340086	0.2519088490098112	5.131488959567799
23	-8.474518880915847	0.23701113601218063	5.026128443909973
24	-11.937130030043246	-1.3148371040020026	4.920767928252147
25	-14.007248684702276	-4.429207501722215	4.81540741259432
26	-16.85664921608761	-6.05823799616474	4.710046896936493
27	-18.305108212486793	-6.312339379469375	4.604686381278667
28	-20.67852084119137	-10.107157656928964	4.4993258656208415
29	-21.379176811974713	-10.783081335039341	4.393965349963015
30		-10.815571830254825	4.288604834305189
31		-12.751136541735296	4.183244318647363
32		-12.810111636133948	4.077883802989536
33		-12.812103604537558	3.97252328733171
34		-22.24479189931832	3.8671627716738834

Matrix 7 Results

Matrix A:

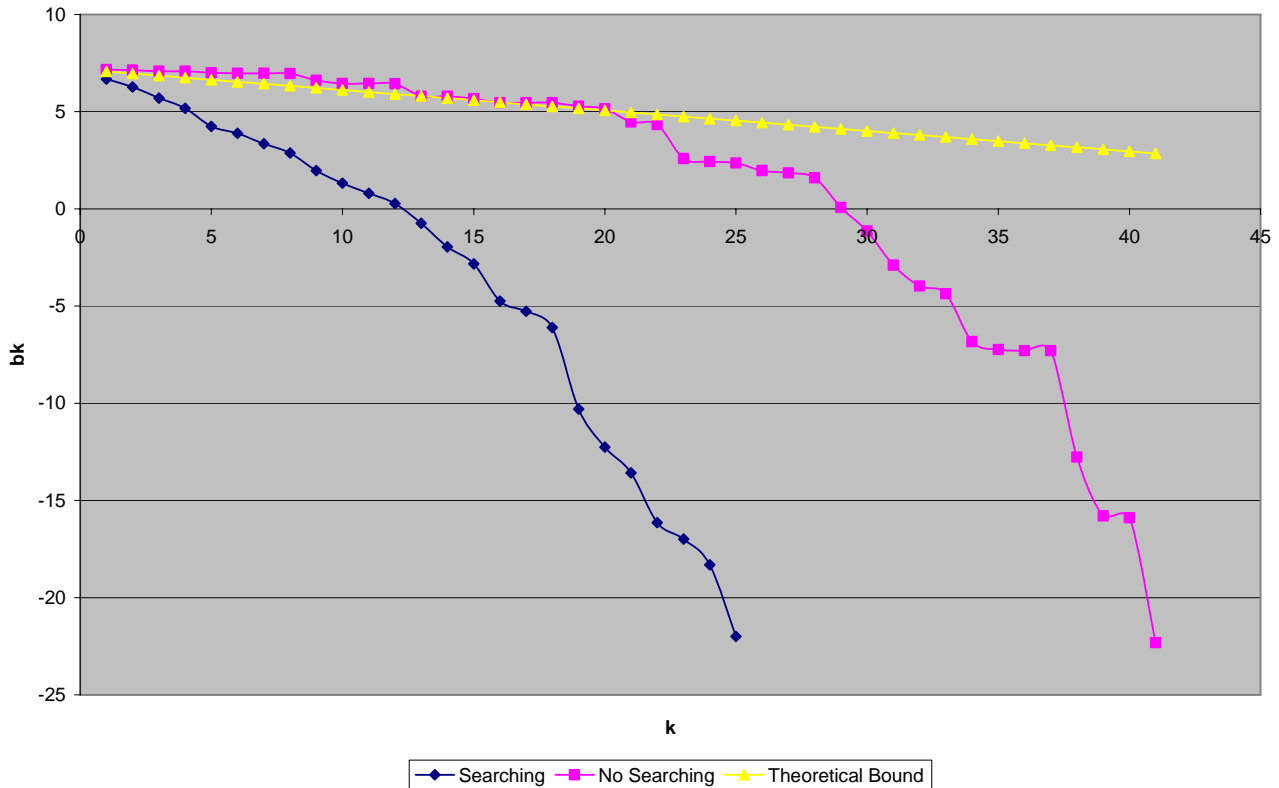
$$\begin{bmatrix} -15.44 & -0.54 & -11.41 & -10.69 & -7.97 \\ -0.54 & -5.95 & -3.63 & 3.99 & 15.95 \\ -11.41 & -3.63 & 16.62 & -1.81 & -3.99 \\ -10.69 & 3.99 & -1.81 & -10.82 & -6.16 \\ -7.97 & 15.95 & -3.99 & -6.16 & 1.14 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 21.91 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 14.29 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -2.88 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -15.71 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -32.05 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 25	Total Iterations: 41
Total Time: < 0.00001 seconds	Total Time: 0.0010 seconds

Jacobian Matrix Comparison 7



Matrix 7 Data

k	$b_k(\text{Searchching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	6.6707075874613135	7.163430734017684	7.063114968110601
2	6.270148032345049	7.143445948163787	6.957754452452774
3	5.697516912639867	7.07955562834769	6.8523939367949485
4	5.172826834659315	7.079067352639239	6.747033421137122
5	4.239217591924486	7.000550882255126	6.641672905479296
6	3.8787272609740797	6.9757398332596585	6.536312389821469
7	3.35015936464117	6.9721522506759595	6.430951874163643
8	2.868847811137359	6.961796958090815	6.325591358505816
9	1.9621488793447972	6.611757826790596	6.22023084284799
10	1.3190451557237712	6.449530679446443	6.1148703271901645
11	0.7915400393344374	6.443624125610616	6.009509811532338
12	0.27661606058510846	6.434031111873611	5.904149295874512
13	-0.7459509239511343	5.789360441929289	5.798788780216685
14	-1.9509827619387792	5.788130058777782	5.693428264558859
15	-2.8286449642309224	5.658983470294269	5.588067748901032
16	-4.742728489029026	5.463985445571608	5.4827072332432065
17	-5.266949022370339	5.46134893589985	5.377346717585381
18	-6.111125873144227	5.453325347787258	5.271986201927554
19	-10.307224892532464	5.275568362065173	5.166625686269727
20	-12.261648473244941	5.146202543720045	5.061265170611901
21	-13.574428061608026	4.472551728222706	4.955904654954075
22	-16.139989159166653	4.336270947433569	4.8505441392962485
23	-16.993974132755138	2.5795089270362803	4.745183623638423
24	-18.308469925718683	2.4274753644736875	4.639823107980597
25	-21.997776270917424	2.3622010650945424	4.53446259232277
26		1.9706172788610994	4.429102076664943
27		1.84282576268093	4.323741561007117
28		1.5898304857339567	4.218381045349291
29		0.06802309442137258	4.113020529691465
30		-1.1514114679229024	4.007660014033639
31		-2.9021187590635766	3.9022994983758124
32		-3.975411801821054	3.796938982717986
33		-4.3679422829433525	3.6915784670601597
34		-6.8369740410404924	3.5862179514023333
35		-7.242849660698892	3.480857435744507
36		-7.293114277842648	3.3754969200866807
37		-7.296583794505208	3.270136404428855
38		-12.772967259663709	3.164775888771028
39		-15.79342144268651	3.059415373113202
40		-15.886658689431632	2.9540548574553753
41		-22.322506516524907	2.8486943417975494

Matrix 8 Results

Matrix A:

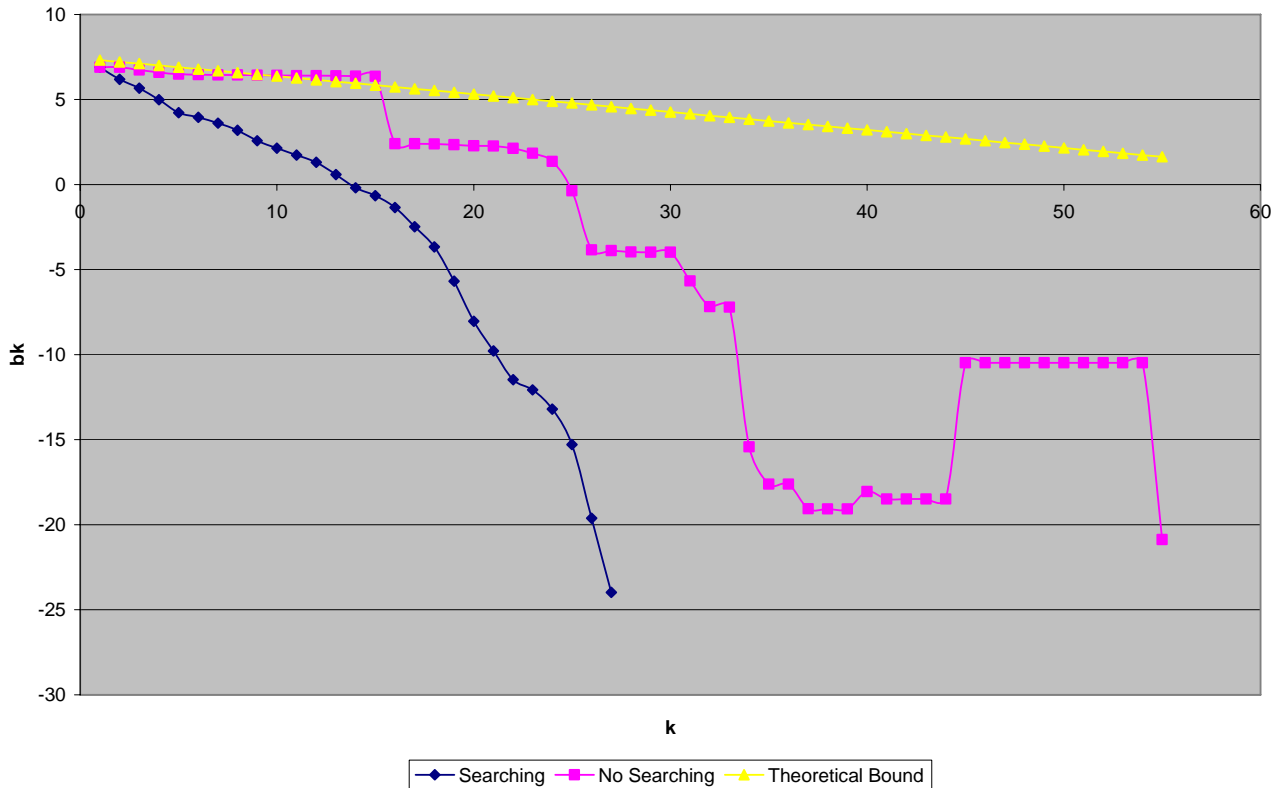
$$\begin{bmatrix} -16.70 & -10.56 & -12.10 & -6.90 & -0.57 \\ -10.56 & 10.62 & -18.55 & -6.08 & 4.54 \\ -12.10 & -18.55 & 16.95 & -4.02 & -7.25 \\ -6.90 & -6.08 & -4.02 & 8.92 & -7.81 \\ -0.57 & 4.54 & -7.25 & -7.81 & 14.94 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 36.71 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 17.66 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 7.06 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 3.82 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -30.53 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 27	Total Iterations: 55
Total Time: 0.0010 seconds	Total Time: 0.0010 seconds

Jacobian Algorithm Comparison 8



Matrix 8 Data

k	$b_k(\text{Searching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	6.893803917101603	6.893803917101603	7.317883504446058
2	6.178678945894858	6.89233272375808	7.212522988788232
3	5.67060638837928	6.723578212442389	7.107162473130406
4	4.988783197764727	6.589000268906747	7.001801957472579
5	4.218835549729171	6.488131683101457	6.896441441814753
6	3.9560951118530703	6.464295867559907	6.791080926156926
7	3.6129518551905364	6.448113169974914	6.6857204104991
8	3.189836594487736	6.43787093487429	6.5803598948412745
9	2.581279857254204	6.432302751202027	6.474999379183448
10	2.133745003660136	6.410502706166407	6.369638863525621
11	1.727830823815946	6.406497018077306	6.264278347867795
12	1.3045366338355375	6.405474311271343	6.158917832209969
13	0.5904748873048582	6.394609600781043	6.053557316552142
14	-0.1863881971702039	6.378096573263138	5.9481968008943165
15	-0.6601635887454748	6.36887296751084	5.842836285236491
16	-1.3536868827100983	2.392406137441128	5.737475769578664
17	-2.480118662075654	2.39117093373638	5.632115253920837
18	-3.66702651483209	2.3901256080605746	5.526754738263011
19	-5.687082373152686	2.347763590705076	5.421394222605185
20	-8.044252286908682	2.2708346965794077	5.316033706947358
21	-9.780173521370482	2.2677538318210715	5.2106731912895325
22	-11.47811285046454	2.1286253884944055	5.105312675631707
23	-12.073599153793264	1.845505815456888	4.99995215997388
24	-13.198712366504703	1.3614321210213751	4.894591644316053
25	-15.283871541385226	-0.36677323323206623	4.789231128658227
26	-19.622199597615793	-3.8446667201913267	4.683870613000401
27	-23.97804132192826	-3.888184829160413	4.5785100973425745
28		-3.9598711122959673	4.473149581684749
29		-3.975278321557024	4.367789066026923
30		-3.975535553632089	4.262428550369096
31		-5.67085678694372	4.157068034711269
32		-7.174456402207961	4.051707519053443
33		-7.208323023328362	3.946347003395617
34		-15.418871312433778	3.8409864877377906
35		-17.60938133992866	3.7356259720799643
36		-17.621819369515233	3.630265456422138
37		-19.061657800764994	3.524904940764312
38		-19.07742263756104	3.4195444251064853
39		-19.077337678162472	3.3141839094486594
40		-18.047312498731277	3.2088233937908326
41		-18.48883596406151	3.1034628781330067
42		-18.4888555766606	2.998102362475181
43		-18.4888555766606	2.892741846817354
44		-18.4888555766606	2.787381331159528

45		-10.48168564782304	2.6820208155017013
46		-10.481654783516372	2.5766602998438755
47		-10.481654783516372	2.4712997841860487
48		-10.481654783516372	2.365939268528223
49		-10.481654798190558	2.260578752870397
50		-10.481987887971957	2.15521823721257
51		-10.481987887971957	2.049857721554744
52		-10.481987887971957	1.9444972058969174
53		-10.481987887971957	1.8391366902390915
54		-10.481987887971957	1.7337761745812648
55		-20.867362332805254	1.6284156589234389

Matrix 9 Results

Matrix A:

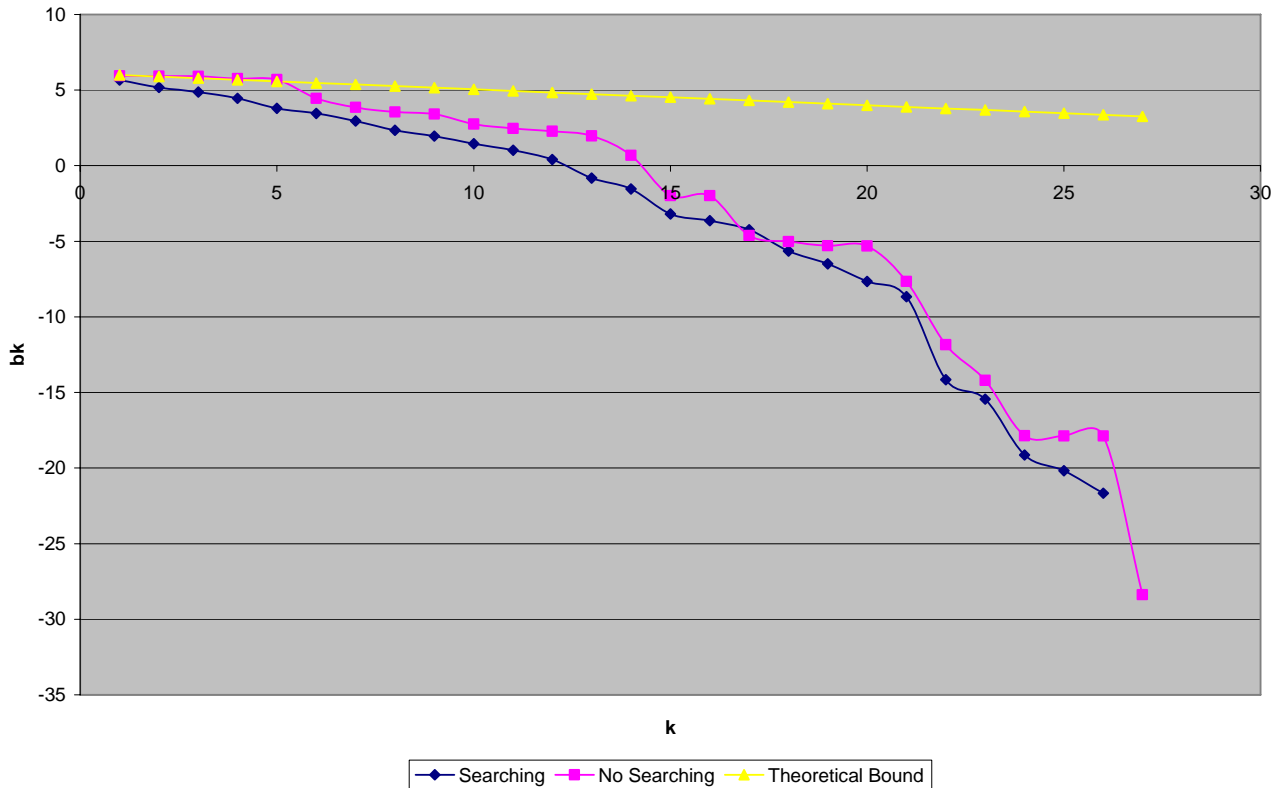
$$\begin{bmatrix} 18.95 & 4.85 & -3.00 & 0.33 & 2.60 \\ 4.85 & -2.27 & -5.83 & 4.26 & -6.22 \\ -3.00 & -5.83 & 10.12 & 3.43 & -1.51 \\ 0.33 & 4.26 & 3.43 & -17.87 & 8.90 \\ 2.60 & -6.22 & -1.51 & 8.90 & -19.15 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 21.70 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 10.52 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & -2.24 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -9.99 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -30.22 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 26	Total Iterations: 27
Total Time: < 0.00001 seconds	Total Time: 0.0010 seconds

Jacobian Algorithm Comparison 9



Matrix 9 Data

k	$b_k(\text{Searchching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	5.664575682181738	5.937474067538639	5.997076212767655
2	5.178856213418446	5.923485967738479	5.891715697109828
3	4.868838897063252	5.913555766074839	5.786355181452002
4	4.458911720586855	5.7546356846856135	5.6809946657941754
5	3.7901206149123277	5.690694315762289	5.5756341501363496
6	3.454194225076764	4.457073177427325	5.470273634478523
7	2.9490861122728305	3.846463610271547	5.364913118820697
8	2.343679759908137	3.544443004517017	5.25955260316287
9	1.9545355397274915	3.4072716385500557	5.154192087505044
10	1.4549466218275946	2.745867364012028	5.048831571847218
11	1.0133543472247715	2.454362036525968	4.9434710561893915
12	0.41046406736446656	2.280002980587128	4.838110540531566
13	-0.822048485664311	1.9647597253781997	4.732750024873739
14	-1.5359018273314138	0.6804514059457989	4.627389509215913
15	-3.1996005524784135	-1.9837062333469544	4.522028993558086
16	-3.6353649605421414	-1.9894600456602305	4.41666847790026
17	-4.239315087451544	-4.620514368202608	4.311307962242434
18	-5.646526142300854	-5.018753569525559	4.205947446584608
19	-6.494930883791339	-5.292792488788889	4.100586930926781
20	-7.654249098361071	-5.305868830440338	3.995226415268955
21	-8.662630972459104	-7.659608662606092	3.889865899611129
22	-14.160542268516142	-11.845707924211538	3.7845053839533027
23	-15.44767993686139	-14.195130726974906	3.6791448682954764
24	-19.137830483250397	-17.87077466827604	3.57378435263765
25	-20.1849898388359	-17.872257765492456	3.4684238369798237
26	-21.671883978308184	-17.873874366470126	3.3630633213219974
27		-28.379989155175885	3.257702805664171

Matrix 10 Results

Matrix A:

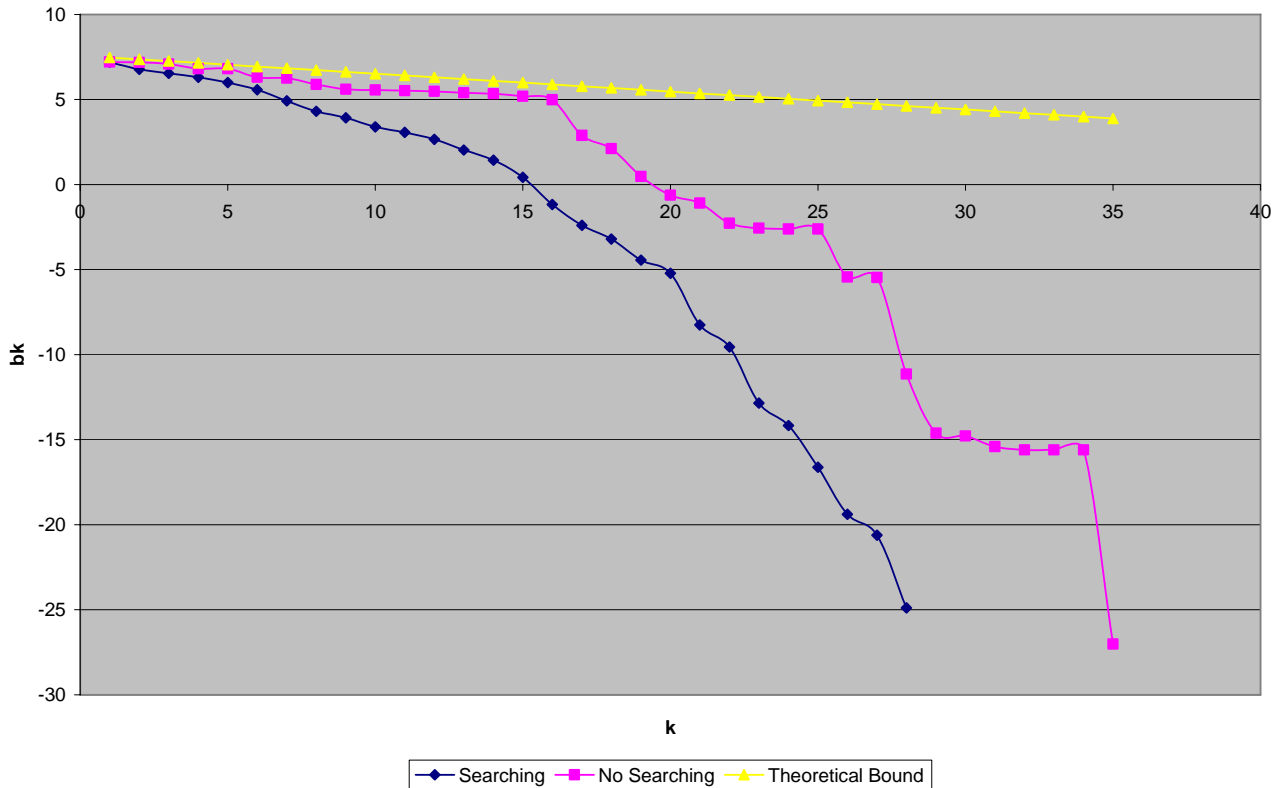
$$\begin{bmatrix} -1.21 & 17.12 & 2.62 & 17.51 & 2.80 \\ 17.12 & -2.34 & 4.00 & -0.66 & 15.17 \\ 2.62 & 4.00 & 4.50 & -10.01 & 3.01 \\ 17.51 & -0.66 & -10.01 & -1.41 & -0.55 \\ 2.80 & 15.17 & 3.01 & -0.55 & 6.40 \end{bmatrix}$$

Diagonal Matrix:

$$\begin{bmatrix} 27.80 & 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 16.88 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.00 & 1.90 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & -12.45 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & -28.20 \end{bmatrix}$$

Searching	No Searching
Total Iterations: 28	Total Iterations: 35
Total Time: < 0.00001 seconds	Total Time: < 0.00001 seconds

Jacobian Algorithm Comparison 10



Matrix 10 Data

k	$b_k(\text{Searching})$	$b_k(\text{No Searching})$	$b_k(\text{Theoretical})$
1	7.19128449545256	7.211633225670481	7.465754825179671
2	6.765837348860926	7.17903379663885	7.360394309521844
3	6.536725072100122	7.079275904981142	7.255033793864018
4	6.3098994629795895	6.80993780056332	7.149673278206191
5	5.998133407559262	6.807792646203148	7.0443127625483655
6	5.577768379048095	6.312301803399827	6.938952246890539
7	4.911526063051266	6.259423962849935	6.833591731232713
8	4.300958066935046	5.883275843268547	6.728231215574887
9	3.928080604439871	5.602315280135594	6.62287069991706
10	3.392096950123785	5.5568684772093455	6.517510184259233
11	3.061799109525106	5.517999413483389	6.4121496686014074
12	2.6550950930966337	5.471291369872765	6.3067891529435816
13	2.0233729544474843	5.39007635830045	6.201428637285755
14	1.4272381213964347	5.340376494207896	6.096068121627929
15	0.4231164195701124	5.19370625256287	5.990707605970103
16	-1.174771064881664	4.99293287161278	5.885347090312276
17	-2.3911937708844366	2.882569650540427	5.779986574654449
18	-3.197840431799584	2.1120881165088967	5.6746260589966235
19	-4.435489229671637	0.47643040569763134	5.569265543338798
20	-5.219281169934487	-0.6328841283463248	5.463905027680971
21	-8.253246677581044	-1.0909953819114457	5.358544512023145
22	-9.543909901677576	-2.2805586236395063	5.253183996365319
23	-12.846557386114782	-2.564480585458338	5.147823480707492
24	-14.17257700175007	-2.6123556415003932	5.0424629650496655
25	-16.616339628668012	-2.620130901945384	4.93710244939184
26	-19.3801706141235	-5.433413400334144	4.831741933734014
27	-20.619557878359956	-5.463240702713021	4.726381418076187
28	-24.89107187900672	-11.150013047715888	4.621020902418361
29		-14.623226026962872	4.515660386760535
30		-14.775457565237321	4.410299871102708
31		-15.411451708187329	4.304939355444882
32		-15.595775591164426	4.199578839787056
33		-15.595843981905139	4.09421832412923
34		-15.595843981905139	3.988857808471403
35		-27.01347841397362	3.8834972928135767